

# Contents

---

Preface .....	vii
1. The Basics .....	1
1.1 Graphs* .....	2
1.2 The degree of a vertex* .....	5
1.3 Paths and cycles* .....	6
1.4 Connectivity* .....	10
1.5 Trees and forests* .....	13
1.6 Bipartite graphs* .....	17
1.7 Contraction and minors* .....	19
1.8 Euler tours* .....	22
1.9 Some linear algebra .....	23
1.10 Other notions of graphs .....	28
Exercises .....	30
Notes .....	33
2. Matching Covering and Packing .....	35
2.1 Matching in bipartite graphs* .....	36
2.2 Matching in general graphs <sup>(*)</sup> .....	41
2.3 Packing and covering .....	45
2.4 Tree-packing and arboricity .....	48
2.5 Path covers .....	52
Exercises .....	54
Notes .....	56

---

\* Sections marked by an asterisk are recommended for a first course.

Of sections marked <sup>(\*)</sup>, the beginning is recommended for a first course.

3. Connectivity .....	59
3.1 2-Connected graphs and subgraphs* .....	59
3.2 The structure of 3-connected graphs <sup>(*)</sup> .....	62
3.3 Menger's theorem* .....	66
3.4 Mader's theorem .....	72
3.5 Linking pairs of vertices <sup>(*)</sup> .....	74
Exercises .....	82
Notes .....	85
4. Planar Graphs .....	87
4.1 Topological prerequisites* .....	88
4.2 Plane graphs* .....	90
4.3 Drawings .....	96
4.4 Planar graphs: Kuratowski's theorem* .....	100
4.5 Algebraic planarity criteria .....	105
4.6 Plane duality .....	107
Exercises .....	111
Notes .....	114
5. Colouring .....	117
5.1 Colouring maps and planar graphs* .....	118
5.2 Colouring vertices* .....	120
5.3 Colouring edges* .....	125
5.4 List colouring .....	127
5.5 Perfect graphs .....	132
Exercises .....	139
Notes .....	143
6. Flows .....	145
6.1 Circulations <sup>(*)</sup> .....	146
6.2 Flows in networks* .....	147
6.3 Group-valued flows .....	150
6.4 $k$ -Flows for small $k$ .....	155
6.5 Flow-colouring duality .....	158
6.6 Tutte's flow conjectures .....	161
Exercises .....	165
Notes .....	167

7. Extremal Graph Theory .....	169
7.1 Subgraphs* .....	170
7.2 Minors <sup>(*)</sup> .....	175
7.3 Hadwiger's conjecture* .....	178
7.4 Szemerédi's regularity lemma .....	182
7.5 Applying the regularity lemma .....	189
Exercises .....	195
Notes .....	198
8. Infinite Graphs .....	203
8.1 Basic notions, facts and techniques* .....	204
8.2 Paths, trees, and ends <sup>(*)</sup> .....	213
8.3 Homogeneous and universal graphs* .....	222
8.4 Connectivity and matching .....	225
8.5 Graphs with ends: the topological viewpoint .....	235
8.6 Recursive structures .....	248
Exercises .....	251
Notes .....	261
9. Ramsey Theory for Graphs .....	269
9.1 Ramsey's original theorems* .....	270
9.2 Ramsey numbers <sup>(*)</sup> .....	273
9.3 Induced Ramsey theorems .....	276
9.4 Ramsey properties and connectivity <sup>(*)</sup> .....	286
Exercises .....	289
Notes .....	290
10. Hamilton Cycles .....	293
10.1 Sufficient conditions* .....	293
10.2 Hamilton cycles and degree sequences* .....	297
10.3 Hamilton cycles in the square of a graph .....	300
Exercises .....	305
Notes .....	306

11. Random Graphs .....	309
11.1 The notion of a random graph* .....	310
11.2 The probabilistic method* .....	315
11.3 Properties of almost all graphs* .....	318
11.4 Threshold functions and second moments .....	322
Exercises .....	329
Notes .....	330
12. Minors, Trees and WQO .....	333
12.1 Well-quasi-ordering* .....	334
12.2 The graph minor theorem for trees* .....	335
12.3 Tree-decompositions .....	337
12.4 Tree-width and forbidden minors .....	345
12.5 The graph minor theorem <sup>(*)</sup> .....	359
Exercises .....	368
Notes .....	373
A. Infinite sets .....	377
B. Surfaces .....	383
Hints for all the exercises.....	391
Index .....	419
Symbol index .....	435