

# Preface

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Almost two decades have passed since the appearance of those graph theory texts that still set the agenda for most introductory courses taught today. The canon created by those books has helped to identify some main fields of study and research, and will doubtless continue to influence the development of the discipline for some time to come.

Yet much has happened in those 20 years, in graph theory no less than elsewhere: deep new theorems have been found, seemingly disparate methods and results have become interrelated, entire new branches have arisen. To name just a few such developments, one may think of how the new notion of list colouring has bridged the gulf between invariants such as average degree and chromatic number, how probabilistic methods and the regularity lemma have pervaded extremal graph theory and Ramsey theory, or how the entirely new field of graph minors and tree-decompositions has brought standard methods of surface topology to bear on long-standing algorithmic graph problems.

Clearly, then, the time has come for a reappraisal: *what are, today, the essential areas, methods and results that should form the centre of an introductory graph theory course aiming to equip its audience for the most likely developments ahead?*

I have tried in this book to offer material for such a course. In view of the increasing complexity and maturity of the subject, I have broken with the tradition of attempting to cover both theory and applications: this book offers an introduction to the theory of graphs as part of (pure) mathematics; it contains neither explicit algorithms nor ‘real world’ applications. My hope is that the potential for depth gained by this restriction in scope will serve students of computer science as much as their peers in mathematics: assuming that they prefer algorithms but will benefit from an encounter with pure mathematics of *some* kind, it seems an ideal opportunity to look for this close to where their heart lies!

In the selection and presentation of material, I have tried to accommodate two conflicting goals. On the one hand, I believe that an

introductory text should be lean and concentrate on the essential, so as to offer guidance to those new to the field. As a graduate text, moreover, it should get to the heart of the matter quickly: after all, the idea is to convey at least an impression of the depth and methods of the subject. On the other hand, it has been my particular concern to write with sufficient detail to make the text enjoyable and easy to read: guiding questions and ideas will be discussed explicitly, and all proofs presented will be rigorous and complete.

A typical chapter, therefore, begins with a brief discussion of what are the guiding questions in the area it covers, continues with a succinct account of its classic results (often with simplified proofs), and then presents one or two deeper theorems that bring out the full flavour of that area. The proofs of these latter results are typically preceded by (or interspersed with) an informal account of their main ideas, but are then presented formally at the same level of detail as their simpler counterparts. I soon noticed that, as a consequence, some of those proofs came out rather longer in print than seemed fair to their often beautifully simple conception. I would hope, however, that even for the professional reader the relatively detailed account of those proofs will at least help to minimize reading time. . .

If desired, this text can be used for a lecture course with little or no further preparation. The simplest way to do this would be to follow the order of presentation, chapter by chapter: apart from two clearly marked exceptions, any results used in the proof of others precede them in the text.

Alternatively, a lecturer may wish to divide the material into an easy basic course for one semester, and a more challenging follow-up course for another. To help with the preparation of courses deviating from the order of presentation, I have listed in the margin next to each proof the reference numbers of those results that are used in that proof. These references are given in round brackets: for example, a reference (4.1.2) in the margin next to the proof of Theorem 4.3.2 indicates that Lemma 4.1.2 will be used in this proof. Correspondingly, in the margin next to Lemma 4.1.2 there is a reference [4.3.2] (in square brackets) informing the reader that this lemma will be used in the proof of Theorem 4.3.2. Note that this system applies between different sections only (of the same or of different chapters): the sections themselves are written as units and best read in their order of presentation.

The mathematical prerequisites for this book, as for most graph theory texts, are minimal: a first grounding in linear algebra is assumed for Chapter 1.9 and once in Chapter 5.5, some basic topological concepts about the Euclidean plane and 3-space are used in Chapter 4, and a previous first encounter with elementary probability will help with Chapter 11. (Even here, all that is assumed formally is the knowledge of basic definitions: the few probabilistic tools used are developed in the

text.) There are two areas of graph theory which I find both fascinating and important, especially from the perspective of pure mathematics adopted here, but which are not covered in this book: these are algebraic graph theory and infinite graphs.

At the end of each chapter, there is a section with exercises and another with bibliographical and historical notes. Many of the exercises were chosen to complement the main narrative of the text: they illustrate new concepts, show how a new invariant relates to earlier ones, or indicate ways in which a result stated in the text is best possible. Particularly easy exercises are identified by the superscript  $-$ , the more challenging ones carry a  $+$ . The notes are intended to guide the reader on to further reading, in particular to any monographs or survey articles on the theme of that chapter. They also offer some historical and other remarks on the material presented in the text.

Ends of proofs are marked by the symbol  $\square$ . Where this symbol is found directly below a formal assertion, it means that the proof should be clear after what has been said—a claim waiting to be verified! There are also some deeper theorems which are stated, without proof, as background information: these can be identified by the absence of both proof and  $\square$ .

Almost every book contains errors, and this one will hardly be an exception. I shall try to post on the Web any corrections that become necessary. The relevant site may change in time, but will always be accessible via the following two addresses:

<http://www.springer-ny.com/supplements/diestel/>  
<http://www.springer.de/catalog/html-files/deutsch/math/3540609180.html>

Please let me know about any errors you find.

Little in a textbook is truly original: even the style of writing and of presentation will invariably be influenced by examples. The book that no doubt influenced me most is the classic GTM graph theory text by Bollobás: it was in the course recorded by this text that I learnt my first graph theory as a student. Anyone who knows this book well will feel its influence here, despite all differences in contents and presentation.

I should like to thank all who gave so generously of their time, knowledge and advice in connection with this book. I have benefited particularly from the help of N. Alon, G. Brightwell, R. Gillett, R. Halin, M. Hintz, A. Huck, I. Leader, T. Łuczak, W. Mader, V. Rödl, A.D. Scott, P.D. Seymour, G. Simonyi, M. Škovič, R. Thomas, C. Thomassen and P. Valtr. I am particularly grateful also to Tommy R. Jensen, who taught me much about colouring and all I know about  $k$ -flows, and who invested immense amounts of diligence and energy in his proofreading of the preliminary German version of this book.

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